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Estimation of heat source term in three-dimensional heat conduction problem from temperature measurements

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Abstract: Estimation of heat source term in three-dimensional heat conduction problem from temperature measurements is a typical Inverse Heat Conduction Problem (IHCP). In this paper, based on the numerical simulation of three-dimensional steady heat conduction problem with the Finite Volume Method (FVM), this three-dimensional IHCP is converted to an optimization problem, and the inversion method for estimation of heat source term is developed from sensitivity analysis. After applying this method to a typical test case, it can be seen that the estimation method is feasible and is not sensitive to the measurement noise. Moreover, the role of the stop criterion in the estimation process of cases with measurement noise is investigated. By analyzing the change of difference between the estimated heat sources and the exact values along with the decreasing of the objective function, it is shown that the decreasing of the objective function does not certainly mean the closer agreement between the estimated heat sources and exact values due to the ill-posedness of the IHCP. This ill-posedness can be overcome by choosing a proper specification of stop criterion to simulate the regularization effect in IHCP.

Key words: three dimensional inverse heat conduction problem; temperature measurement; heat source term; stop criterion

利用温度测量结果反演三维热传导问题中热源项的算法研究

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摘要:利用表面温度测量来反演热传导问题中的热源项是一类典型的热传导逆问题,在采用有限体积法对三维稳态热传导问题进行数值求解的基础上,将该热传导逆问题转化为优化问题,基于灵敏度分析建立了反演算法。采用该算法对一典型算例的计算结果表明:建立的算法是有效的,具有较好的抗噪性能。此外,对反演算法中计算收敛准则的选取进行了较深入的分析,结果表明,由于热传导逆问题的不适定性,优化过程中目标函数值越小并不意味着反演结果与真值更为接近,可以通过设定合适的收敛准则来模拟正则化项的作用,克服不适定性的影响。

关键词:三维热传导逆问题;温度测量;热源项;收敛准则

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0 Introduction

The direct or standard heat conduction problems are concerned with the determination of temperature distribution in the interior of a solid when the initial and boundary conditions, thermo-physical properties, and heat sources are specified. In contrast, the Inverse Heat Conduction Problem (IHCP) involves the determination of the surface condition, thermo-properties, or heat source term by utilizing the measured temperature history at one or more locations in the solid. The IHCP has numerous applications in many fields of science and engineering, such as astronautic, nuclear physical, and metallurgical research fields[1]. For example, the estimation of surface heat flux in the quenching process of material is crucial in metallurgy industry. And the inversion of surface aerodynamic heating flux is an effective method to estimate the aero-thermal environment of the reentry vehicle in the reentry phase[2]. However, many available approaches to IHCP are dealing with one or two dimensional cases which inherently involve great discrepancies from the engineering practical situation[1-5] and the discussion of three-dimensional IHCP is very limited in the literature. So, in this paper, based on solving the three-dimensional heat conduction equation with Finite Volume Method (FVM) which is prevailing in Computational Fluid Dynamics (CFD) applications, the IHCP of estimating distributed heat source term in three-dimensional heat conduction problem from temperature measurements is investigated.

1 Three-dimensional direct heat conduction problem

The three-dimension direct heat conduction problem is sketched in Fig.1. The research object is a thin cuboid with length (x direction) of 1, wideness (y direction) of 1, and thickness (z direction) of 0.05. The selection of the small value of thickness is to avoid the influence of dampness in heat conduction process. The cuboid is composed of two parts Ω_1 and Ω_2, whose thickness is 0.02 and 0.03 respectively. These two parts have same heat conduction coefficient but have different heat sources. The heat source in Ω_1 is 0 while in Ω_2 the heat source is $g(\Omega_2)$.

After applying the third-class and the first-class boundary condition on the sides of $x = 0$ and $x = 1$ of the cuboid respectively, and employing the third-class boundary condition on the both sides in y direction and the second-class boundary condition on the both sides in z direction. The heat conduction equation can be written as

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z}\right) + g(\Omega_2)\delta(\Omega_i - \Omega_2) = 0 \tag{1}$$

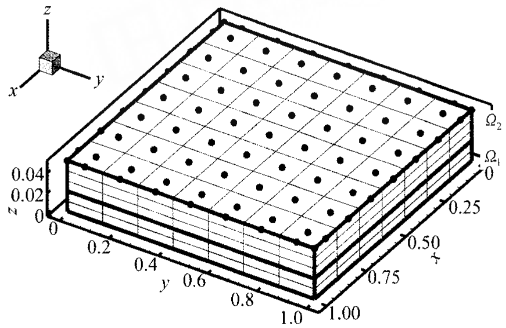

Fig.1 Sketch of 3D heat conduction problem
图 1 三维热传导问题示意图

with boundary conditions

$$x = 0,\ \frac{\partial T}{\partial x} = h(T - T_\infty);\ x = 1,\ T = 1;$$

$$y = 0,\ \frac{\partial T}{\partial y} = h(T - T_\infty);$$

$$y = 1,\ \frac{\partial T}{\partial y} = -h(T - T_\infty);$$

$$z = 0,\ \frac{\partial T}{\partial z} = -1;\ z = 0.05,\ \frac{\partial T}{\partial z} = 0$$

where k, h and T_∞ are constants.

This equation can be solved with Finite Volume Method (FVM) which is prevailing in Computational Fluid Dynamics (CFD) applications[6]. At first, the cuboid is divided into $7 \times 7 \times 5$ finite volumes, and a pseudo time of τ is introduced into Eq.(1) and turns it into

$$\frac{\partial T}{\partial \tau} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + g(\Omega_2)\delta(\Omega_i - \Omega_2) \tag{2}$$

Using explicit fourth-order Runge-Kutta scheme to integrate the temporal derivative in this equation and discretizing the spatial derivatives with second-order central difference scheme, Eq.(2) can be solved with time marching technique and then the resulting convergent solution is the corresponding solution of Eq.(1).

2 Inverse heat conduction problem

As mentioned above, there are many types of IHCP such as estimation of exterior surface heat or temperature from some interior measurements, or estimation of the material's heat conduction coefficient from measured temperature history at one or more locations in the solid. In this paper, the IHCP of estimating distributed heat source term in three-dimensional heat conduction problem from temperature measurements is concerned. That is to estimate the heat source term of $g(\Omega_2)$ in Eq.(1) from the temperature measurements of M locations on the solid surface of $z = 0.05$ in Fig.1. The value of M is 81 and all the M locations of measurement are shown scattered in Fig.1.

The solution of this IHCP is to be obtained in such a way that the following functional is minimized for $g(\Omega_2)$

$$J(g) = \frac{1}{2}\sum_{m=1}^{M}[T(x_m, y_m, g) - \tilde{T}(x_m, y_m)]^2 \quad (3)$$

here, $T(x_m, y_m)$ are the computed temperature at the measurement locations by solving the Eq.(1) with an estimated value of g, and $\tilde{T}(x_m, y_m)$ is the measured temperature at the measurement locations. Because $g(\Omega_2)$ is a function of spatial locations, it can be discretized due to the FVM approach as

$$g_{ijl} = g(x_i, y_j, z_l)$$
$$i = 1, N_i, \; j = 1, N_j, \; l = 1, N_l \quad (4)$$

where N_i, N_j and N_l are the grid numbers in the x, y, z direction of Ω_2. So, Eq.(3) turns into a parameter optimization problem and can be solved with gradient method. The update process in the gradient method is

$$\hat{g}_{ijl}^{n+1} = \hat{g}_{ijl}^{n} - \lambda\left(\frac{\partial J}{\partial g_{ijl}}\right) \quad (5)$$

where the hat of "^" on g denotes the estimated quantities of heat source terms; the superscripts n, $n+1$ indicate the iteration step; λ is the optimization step size, which is obtained from one-dimension optimization algorithm, such as the Golden Section Method[6]. And the gradient is calculated as

$$\frac{\partial J}{\partial g_{ijl}} = \sum_{m=1}^{M}\left\{[T(x_m, y_m, g) - \tilde{T}(x_m, y_m)]\frac{\partial T}{\partial g_{ijl}}\right\} \quad (6)$$

here, the derivative of $\partial T/\partial g_{ijl}$ is called sensitivity, and it can be obtained by solving the sensitivity equation which is derived by differentiating the Eq.(1) with the parameters, i.e., the sensitivity $U = \partial T/\partial g_{ijl}$ satisfied the following equation

$$k\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z}\right) + \delta(x - x_i)\delta(y - y_j)\delta(z - z_l) = 0 \quad (7)$$

with boundary conditions

$$x = 0, \; \frac{\partial U}{\partial x} = hU; \; x = 1, \; U = 0;$$
$$y = 0, \; \frac{\partial U}{\partial y} = hU; y = 1, \; \frac{\partial U}{\partial y} = -hU;$$
$$z = 0, \; \frac{\partial U}{\partial z} = 0; z = 0.05, \; \frac{\partial U}{\partial z} = 0$$

This equation also can be solved with FVM.

So, the computational procedure for the solution of this IHCP can be summarized as follows

(Step 1) Suppose $\hat{g}_{ijl}^{n}$ is available at n-th time step;

(Step 2) Solve the direct problem (Eq.(1)) for $T(x_m, y_m)$ and calculate objective function J in Eq.(3);

(Step 3) Examine the stop criterion for J and continue if not satisfied;

(Step 4) Solve the sensitivity equations (Eq.(7)) and compute the gradient in Eq.(6);

(Step 5) Update the heat source term with Eq.(5) and return to Step 2.

3 Test cases and discussion

In the test case, the geometry in Fig.1 and the heat conduction problem of Eq.(1) is studied firstly. The parameters are chosen as $k = 1$, $h = 1$, $T_\infty = 0$, respectively, and the heat source term is specified to be a function of surface positions x and y only (not a function of z) as following

$$g_{ijl} = \begin{cases} 4 \times | 20 - (8 - j)^2 - (3 - i)^2 + 5 |; \\ \quad i \le 3, j \le 3, l = 1, N_l \\ 3 \times | 17 - (2 - j)^2 - (7 - i)^2 + 10 |; \\ \quad \text{else} \end{cases} \quad (8)$$

This heat source term function is sketched in Fig.2. Using these parameters and functions, the direct heat conduction problem of Eq.(1) can be solved and the temperature values at the M measurement locations can be obtained. In order to validate the effectiveness of the estimation method for the IHCP, these computed temperatures are used as measurement data to estimate the heat source term. The estimated result of $\hat{g}_{ijl}$ is shown in Fig.3. It can be seen from Fig.2 and Fig.3 that the estimated result is nearly identical to the specified value of heat source, which demonstrates that the estimation method developed for IHCP is effective.

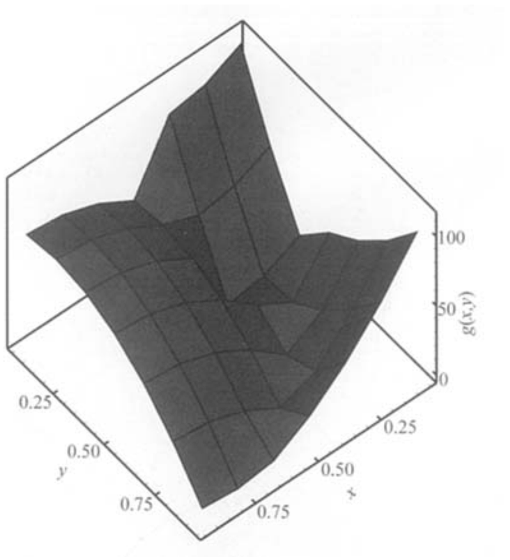

Fig.2 Specified function of heat source term

图2 给定的热源值空间函数

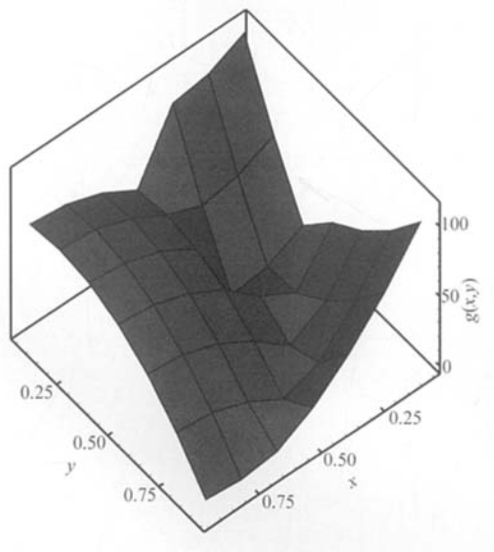

Fig.3 Estimated function of heat source term

图3 反演出的热源值空间函数

Secondly, in order to analyze the influence of the measurement noise to the estimation result, the computed temperatures from specified heat source term in Eq.(8) are now added with some white noise to simulate the measurement data and used to estimate the same heat source term also. Here, two levels of noise whose standard deviations are $\sigma = 0.02$ and $\sigma = 0.1$ are considered. And the conventional stop criterion for optimization as

$$J \leqslant \varepsilon;\ \varepsilon \text{ is a small number} \tag{9}$$

is not adopted in the estimation iteration because of the ill-posedness of the IHCP. The reason lies in that nearly all the IHCP are ill-posed problem, so in the solution of IHCP some regularization terms should be added as "stable functional" in the objective functional to convert the ill-posed problem to be well-posed[7]. But in Eq.(3) there is no regularization terms considered, so, inspired by the idea of iterative regularization proposed by Alifanov[8,9], the following stop criterion instead of Eq.(9) is employed.

$$J \leqslant \delta;\ \delta = M\sigma^2 \tag{10}$$

where M is the number of measurement locations and σ is the standard deviations of measurement noise. Then, based on this stop criterion and the estimation method for IHCP, the test cases for $\sigma = 0.02$ and $\sigma = 0.1$ are studied respectively. The estimated heat source terms of these two cases are shown in Fig.4 and Fig.5 (denoted as dashed lines) and compared to the specified value (denoted as solid lines). In addition, the temperatures at several measurement locations along the grid lines of $i = 2, 7, 8$ calculated with estimated value of heat source term are compared with the measurement data for these two cases in Fig.6 and Fig.7. From these results, it can be seen that the estimated

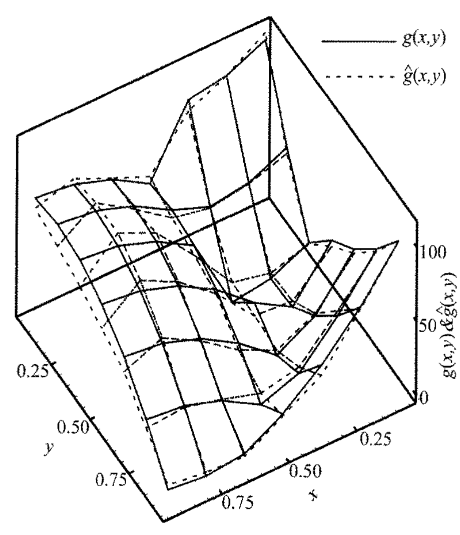

Fig.4 Comparison of estimated and specified value of heat source ($\sigma = 0.02$)

图4 反演结果与热源给定值比较($\sigma = 0.02$)

values of heat source term agree well with the exact values when the measurement noise is considered, and the temperatures calculated with estimated heat source term also fit the measurement data. It demonstrates that the estimation method of IHCP is robust and its estimated result is not greatly deteriorated by the measurement noise.

Finally, the stop criterion in Eq.(10) is further discussed. At first, the difference between the estimated value and exact value of heat source term is defined as

$$E = \sqrt{\sum_{i=1}^{N_i}\sum_{j=1}^{N_j}(\hat{g}_{ij} - g_{ij})^2/(N_i \times N_j)} \qquad (11)$$

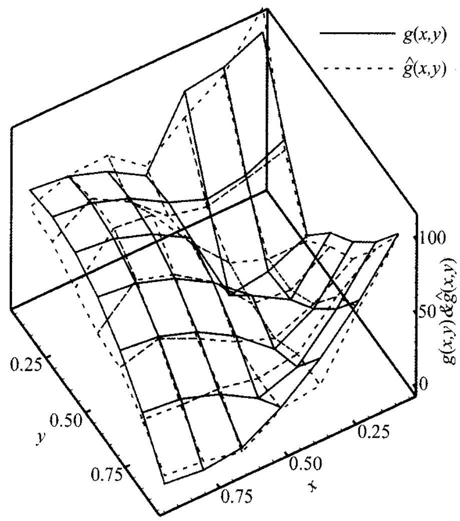

Fig.5 Comparison of estimated and specified value of heat source($\sigma = 0.1$)

图 5　反演结果与热源给定值比较($\sigma = 0.1$)

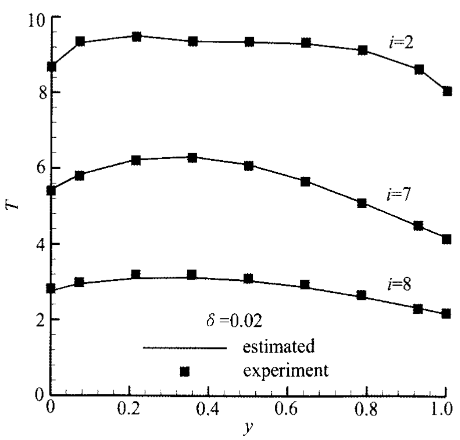

Fig.6 Comparison of temperature calculated with estimated heat source and experimental value ($\sigma = 0.02$)

图 6　利用反演结果计算出的测温点温度分布与实测值比较($\sigma = 0.02$)

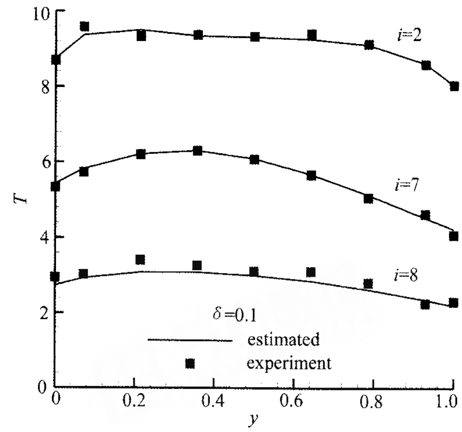

Fig.7 Comparison of temperature calculated with estimated heat source and experimental value ($\sigma = 0.1$)

图 7　利用反演结果计算出的测温点温度分布与实测值比较($\sigma = 0.1$)

Then, still take the aforementioned cases of measurement noise being $\sigma = 0.02$ and $\sigma = 0.1$ as concerned, the respective change of E in the optimization process along with the decreasing of the objective function are shown in Fig.8 and Fig.9 (The arrows indicate the decreasing of the objective function). It can be seen from the figures that:

(1) In both cases, the decreasing of the objective function does not certainly mean the smaller value of E. There exists a minimum value of E in the $E \sim J$ curve, and when the objective function is less than this minimum point, a sharp increase in E comes out (especially in Fig. 9). The reason lies in the ill-posedness of IHCP. The connotation of well-posed solution is composed of three parts, the existence, uniqueness and stability. The solution of IHCP always violates the stability requirement, i.e., a small noise in measurement data may lead to great change in the inversion solution. In order to overcome this ill-posedness, regularization terms have to be added in the objective functional to convert the ill-posed problem to be well-posed. Here, the stop criterion of Eq.(10) is specified to act as a regularization term because the regularization term has been involved implicitly in the value δ. It's just thanks to this criterion that the estimated results in Fig.4 and Fig.5 are not greatly deteriorated when the measurement noise is present.

(2) For both cases there exists a minimum value of E in the $E \sim J$ curve and the inversion result at this mini-

mum point should be the best estimate of the heat source term. Also it can be seen that the minimum point occur near $J = 0.5M\sigma^2$ in both cases. Whether this is an intrinsic characteristic of this IHCP or just a coincidence, which is still a question need to be investigated in the future.

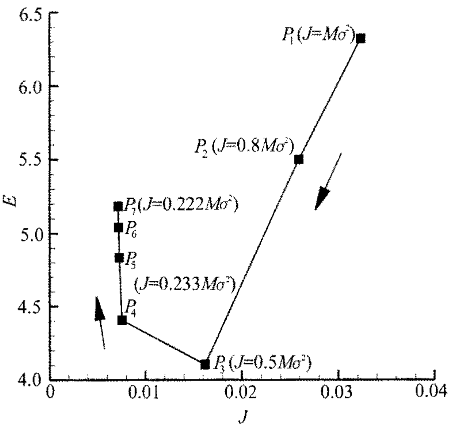

Fig.8 Change of E for decreasing objective function($\sigma = 0.02$)

图 8 E 随目标函数下降的变化过程($\sigma = 0.02$)

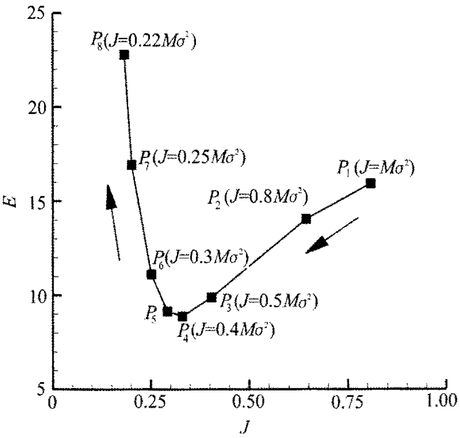

Fig.9 Change of E for decreasing objective function($\sigma = 0.1$)

图 9 E 随目标函数下降的变化过程($\sigma = 0.1$)

4 Conclusion

In this paper, based on the numerical simulation of three-dimensional steady heat conduction problem with the finite volume method (FVM), this three-dimensional IHCP is converted to an optimization problem and the estimation method for heat source term in the heat conduction problem is developed from sensitivity analysis. After applying this method to the IHCP of a thin cuboid, it can be seen that the estimation method is feasible and is not sensitive to the measurement noise. Moreover, the role of the stop criterion in the estimation process of cases in which measurement noise considered is investigated and the change of difference between the estimated heat sources and the exact values along with the decreasing of the objective function is analyzed. It is revealed that when the measurement noise is present, the decreasing of the objective function does not certainly mean the closer agreement between the estimated heat sources and exact values due to the ill-posedness of the IHCP. And this ill-posedness can be compensated by the proper specification of stop criterion to simulate the regularization effect in IHCP.

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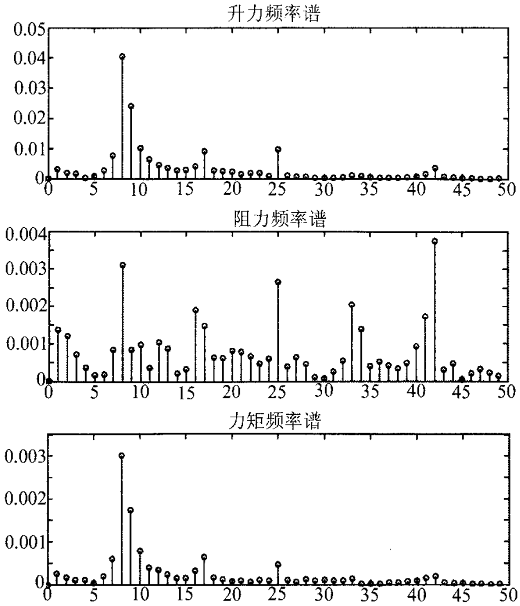

图11 $v=5\text{m/s}, \alpha=4°$状态吹风实验结果频率谱

Fig.11 The spectrum of wind tunnel test in flight status of $v=5\text{m/s}, \alpha=4°$

4 结束语

随着虚拟仪器在测控系统设计领域的大量使用,测量与控制的实时性要求与日俱增。基于安全队列的多线程技术虚拟仪器测控系统符合现代工程软件设计要求,能够满足实时数据采集和控制,而不需要增加设备的附加成本。实验说明,这种技术能够在微型飞行器仿生模型静动态试验中发挥积极可靠的作用。

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